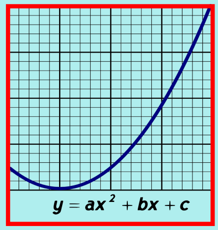
Math 25

Fall 2017

Lecture 5



Solve by Matrix method

$$\begin{cases} x + 2y + z = 0 \\ 2x - y - z = 5 \\ x + y - 2z = 1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & -1 & -1 & 5 \\ 1 & 1 & -2 & 1 \end{array}\right]$$

① Augmented matrix

$(-2)R1 + R2 \to R2$

$(-)R1 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -5 & -3 & 5 \\ 0 & -1 & -3 & 1 \end{array}\right]$$

$R2 \rightleftarrows R3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -1 & -3 & 1 \\ 0 & -5 & -3 & 5 \end{array}\right]$$

$(2)R2 + R1 \to R1$

$(-5)R2 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -1 & -3 & 1 \\ 0 & -5 & -3 & 5 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -5 & 2 \\ 0 & -1 & -3 & 1 \\ 0 & 0 & 12 & 0 \end{array}\right]$$

$R3 \div 12 \rightarrow R3$

$R2 \div (-1) \rightarrow R2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & 2 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$(-3)R3 + R2 \rightarrow R2$

$(5)R3 + R1 \rightarrow R1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} \rightarrow x=2 \\ \rightarrow y=-1 \\ \rightarrow z=0 \end{array}$$

final Ans.

$(2, -1, 0)$

$$A = \begin{bmatrix} 2 & 3 & -2 \\ 0 & 1 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 5 & -2 \\ 4 & 2 & 3 \\ 7 & -1 & -3 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

2×3 $\quad$ 3×3 $\quad$ 2×3

find

$$3A = 3\begin{bmatrix} 2 & 3 & -2 \\ 0 & 1 & -4 \end{bmatrix} = \begin{bmatrix} 6 & 9 & -6 \\ 0 & 3 & -12 \end{bmatrix}$$

find

$$A - 2C = \begin{bmatrix} 2 & 3 & -2 \\ 0 & 1 & -4 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & -2 \\ 0 & 1 & -4 \end{bmatrix} - \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -8 \\ -8 & -9 & -16 \end{bmatrix}$$

Find AB & BA

$$AB = \begin{bmatrix} 2 & 3 & -2 \\ 0 & 1 & -4 \end{bmatrix} \begin{bmatrix} 1 & 5 & -2 \\ 4 & 2 & 3 \\ 7 & -1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 18 & 11 \\ -24 & 6 & 15 \end{bmatrix}$$

2×3 $\qquad$ 3×3

$$BA = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \cdot \begin{bmatrix} & & \\ & & \end{bmatrix}$$

Not possible

3×3 $\qquad$ 2×3

Different

Identity Matrix

1) Square matrix
2) All elements on main diagonal are 1
3) Rest of elements are 0.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If operation can be done.

$IA = A \qquad AI = A$

$2 \times 2 \qquad 2 \times 2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$$

$A = \begin{bmatrix} 2 & -3 & 4 \\ 0 & 1 & 7 \\ 0 & 0 & 2 \end{bmatrix}$ find AI_3

$$\underset{3\times 3}{\begin{bmatrix} 2 & -3 & 4 \\ 0 & 1 & 7 \\ 0 & 0 & 2 \end{bmatrix}} \underset{3\times 3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} = \begin{bmatrix} 2 & -3 & 4 \\ 0 & 1 & 7 \\ 0 & 0 & 2 \end{bmatrix}$$

whenever operation is doable,

If $AB = I$ and $BA = I$, then

A & B are Inverse of each other.

Show A & B are inverse of each other.

$A = \begin{bmatrix} 2 & 9 \\ 1 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 5 & -9 \\ -1 & 2 \end{bmatrix}$

find AB

$$\begin{bmatrix} 2 & 9 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 5 & -9 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

find BA

$$\begin{bmatrix} 5 & -9 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 9 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A & B are inverse of each other.

Inverse of A : A^{-1} → Not exponent

→ Does not mean reciprocal

A must be Square matrix

# of rows = # of columns

$$\left[\begin{array}{c:c} A & I \end{array}\right] \rightarrow \left[\begin{array}{c:c} I & A^{-1} \end{array}\right]$$

Perform elem. row operations

If A does not have an inverse, It is called a singular matrix.

Find A^{-1} if exists for $A = \begin{bmatrix} 2 & 9 \\ 1 & 5 \end{bmatrix}$

$$\left[\begin{array}{cc:cc} 2 & 9 & 1 & 0 \\ 1 & 5 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc:cc} 1 & 5 & 0 & 1 \\ 2 & 9 & 1 & 0 \end{array}\right]$$

$R1 \rightleftarrows R2$ $\qquad$ $(-2)R1 + R2 \rightarrow R2$

$$\left[\begin{array}{cc:cc} 1 & 5 & 0 & 1 \\ 0 & -1 & 1 & -2 \end{array}\right] \rightarrow \left[\begin{array}{cc:cc} 1 & 5 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{array}\right]$$

$(-1)R2 \rightarrow R2$ $\qquad$ $(-5)R2 + R1 \rightarrow R1$

$$\left[\begin{array}{cc:cc} 1 & 0 & 5 & -9 \\ 0 & 1 & -1 & 2 \end{array}\right]$$

I $\qquad$ A^{-1}

$$\boxed{A^{-1} = \begin{bmatrix} 5 & -9 \\ -1 & 2 \end{bmatrix}}$$

$A = \begin{bmatrix} 2 & -5 \\ -6 & 15 \end{bmatrix}$ Find A^{-1}

$$\left[\begin{array}{cc:cc} 2 & -5 & 1 & 0 \\ -6 & 15 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cc:cc} 2 & -5 & 1 & 0 \\ 0 & 0 & 3 & 1 \end{array}\right]$$

$(3)R1 + R2 \to R2$

this element will never becomes 1.

Matrix A is Singular.

A^{-1} does not exist.

$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix}$, find A^{-1}

1) Is A a square matrix?

It is 3x3, ✓

$$\left[\begin{array}{ccc:ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 1 & 1 & -2 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$(-1)R1 + R2 \to R2$

$R1 + R3 \to R3$

$$\left[\begin{array}{ccc:ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 2 & -3 & -1 & 1 & 0 \\ 0 & -1 & 2 & 1 & 0 & 1 \end{array}\right]$$

$R3 + R2 \to R2$

$$\left[\begin{array}{ccc:ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 1 \\ 0 & -1 & 2 & 1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 1 \\ 0 & -1 & 2 & 1 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & -1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 2 \end{array}\right]$$

$R2 + R1 \rightarrow R1$

$R2 + R3 \rightarrow R3$

$R3 + R2 \rightarrow R2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 & 1 & 2 \end{array}\right]$$

$$\underbrace{\quad}_{I_3} \qquad \underbrace{\quad}_{A^{-1}}$$

$$A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 9 \\ 1 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} x \\ y \end{bmatrix} \qquad C = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

Find $AB = \underset{2\times 2}{\begin{bmatrix} 2 & 9 \\ 1 & 5 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{2\times 1}{\begin{bmatrix} 2x+9y \\ x+5y \end{bmatrix}}$

Solve $AB = C$ $\qquad \begin{bmatrix} 2x+9y \\ x+5y \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix} \Rightarrow \begin{matrix} 2x+9y=-3 \\ x+5y=4 \end{matrix}$

Rewrite $\begin{bmatrix} 2 & 9 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 5 & -9 \\ -1 & 2 \end{bmatrix}$

Multiply from the left by A^{-1}

$$AB = C$$
$$\underbrace{A^{-1}A}B = A^{-1}C$$
$$IB = A^{-1}C$$
$$B = A^{-1}C$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & -9 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -51 \\ 11 \end{bmatrix}$$

$\{(-51, 11)\}$

$x = -51$
$y = 11$ $\Rightarrow (-51, 11)$

If $Ax = b$, and A^{-1} exists,

then $A^{-1}Ax = A^{-1}b$

$$Ix = A^{-1}b$$
$$x = A^{-1}b$$

A is the coef. matrix

x is column matrix for variables only

b is column matrix for RHS constants.

$x + y - z = 5$
$2x - y + 4z = 0$
$3x + 2y - z = 1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & -1 & 4 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$$

$$A \cdot x = b$$

If A^{-1} exists, then $\boxed{x = A^{-1}b}$

Solve $\begin{cases} 6x - 2y - z = 5 \\ 4x - 2y + z = 8 \\ -10x + 4y + z = 0 \end{cases}$ by inverse method.

$$\underbrace{\begin{bmatrix} 6 & -2 & -1 \\ 4 & -2 & 1 \\ -10 & 4 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 5 \\ 8 \\ 0 \end{bmatrix}}_{b} \Rightarrow Ax = b$$

If A^{-1} exists,

$x = A^{-1}b$

Looking for A^{-1}

$$\left[\begin{array}{ccc|ccc} 6 & -2 & -1 & 1 & 0 & 0 \\ 4 & -2 & 1 & 0 & 1 & 0 \\ -10 & 4 & 1 & 0 & 0 & 1 \end{array}\right]$$

$(2)R1 \to R1$

$(-3)R2 \to R2$

$R1 + R2 \to R2$

$R1 \div 2 \to R1$

$$\left[\begin{array}{ccc|ccc} 12 & -4 & -2 & 2 & 0 & 0 \\ \cancel{-12}\ 0 & \cancel{6}\ 2 & \cancel{-3}\ -5 & \cancel{0}\ 2 & -3 & 0 \\ -10 & 4 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 6 & -2 & -1 & 1 & 0 & 0 \\ 0 & 2 & -5 & 2 & -3 & 0 \\ -10 & 4 & 1 & 0 & 0 & 1 \end{array}\right]$$

$(5)R1 \to R1$

$(3)R3 \to R3$

$R1 + R3 \to R3$

$R1 \div 5 \to R1$

$$\left[\begin{array}{ccc|ccc} 6 & -2 & -1 & 1 & 0 & 0 \\ 0 & 2 & -5 & 2 & -3 & 0 \\ -10 & 4 & 1 & 0 & 0 & 1 \end{array}\right] \qquad \left[\begin{array}{ccc|ccc} 30 & -10 & -5 & 5 & 0 & 0 \\ 0 & 2 & -5 & 2 & -3 & 0 \\ \cancel{-30}\ 0 & \cancel{12}\ 2 & \cancel{3}\ -2 & \cancel{0}\ 5 & 0 & 3 \end{array}\right]$$

$(5)R1 \to R1$

$(3)R3 \to R3$

$R1 + R3 \to R3$

$R1 \div 5 \to R1$

$$\left[\begin{array}{ccc|ccc} 6 & -2 & -1 & 1 & 0 & 0 \\ 0 & 2 & -5 & 2 & -3 & 0 \\ 0 & 2 & -2 & 5 & 0 & 3 \end{array}\right]$$

$(-1)R2 + R3 \to R3$

$R2 + R1 \to R1$

$R3 \div 3 \to R3$

$$\left[\begin{array}{ccc|ccc} 6 & 0 & -6 & 3 & -3 & 0 \\ 0 & 2 & -5 & 2 & -3 & 0 \\ 0 & 0 & \cancel{3}\ 1 & \cancel{3}\ 1 & \cancel{3}\ 1 & \cancel{3}\ 1 \end{array}\right]$$

$$\left.\begin{array}{ccc|ccc} 6 & 0 & -6 & 3 & -3 & 0 \\ 0 & 2 & -5 & 2 & -3 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 6 & 0 & 0 & 9 & 3 & 6 \\ 0 & 2 & 0 & 7 & 2 & 5 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right]$$

$(5)R3 + R2 \to R2$

$(6)R3 + R1 \to R1$

$R1 \div 6 \to R1$

$R2 \div 2 \to R2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/2 & \frac{1}{2} & 1 \\ 0 & 1 & 0 & 7/2 & 1 & 5/2 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right]$$
$$\underbrace{}_{I} \qquad \underbrace{}_{A^{-1}}$$

$x = A^{-1} b$

$$= \begin{bmatrix} 3/2 & \frac{1}{2} & 1 \\ 7/2 & 1 & 5/2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{23}{2} \\ 51/2 \\ 13 \end{bmatrix}$$

$$\boxed{\left(\frac{23}{2}, \frac{51}{2}, 13\right)}$$

Solve by matrix method:

$$\begin{cases} x - y + z = -4 \\ x + y - 2z = 12 \\ -x + z = -5 \end{cases}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 12 \\ -5 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 1 & 1 & -2 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 1 \\ 0 & -1 & 2 & 1 & 0 & 1 \end{array}\right]$$

$R3 + R2 \rightarrow R2$
$R1 + R3 \rightarrow R3$

$R2 + R1 \rightarrow R1$
$R2 + R3 \rightarrow R3$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 1 \\ 0 & -1 & 2 & 1 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & -1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 2 \end{array}\right]$$

$R3 + R2 \rightarrow R2$

$$\left[\underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I} \;\middle|\; \underbrace{\begin{matrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{matrix}}_{A^{-1}} \right]$$

$x = A^{-1}b$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 12 \\ -5 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 5 \\ -2 \end{bmatrix}$$

$$\boxed{\{(3, 5, -2)\}}$$

Is there a way to determine whether or not A^{-1} exist? Yes

Determinant is a numerical value associated with every square matrix.

If det = 0 $\rightarrow$ No inverse, A is a singular matrix.

How to find the determinant?

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad |A| = ad - bc$ $\quad \begin{vmatrix} 2 & 6 \\ -1 & 4 \end{vmatrix}$

$= 2(4) - 6(-1) = \boxed{14}$

$A = \begin{vmatrix} 7 & 6 \\ 10 & 3 \end{vmatrix} = 7(3) - 10(6) = 21 - 60$

$= \boxed{-39}$

$A = \begin{bmatrix} 5 & 3 \\ -10 & -6 \end{bmatrix}$, $|A| = 5(-6) - 3(-10)$

Matrix $\qquad$ Det. $= -30 + 30$

$= 0$

$\rightarrow$ A is a singular matrix since $|A| = 0$

A^{-1} does not exist.

$$A = \begin{bmatrix} 2 & -1 & 5 \\ 1 & 0 & -4 \\ 3 & -1 & 1 \end{bmatrix}$$

Expand by first row

$$= 2\begin{vmatrix} 0 & -4 \\ -1 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 1 & -4 \\ 3 & 1 \end{vmatrix} + 5\begin{vmatrix} 1 & 0 \\ 3 & -1 \end{vmatrix}$$

$$= 2(0-4) + 1(1 - {}^{-}12) + 5(-1-0)$$

$$= -8 + 13 - 5 = \boxed{0}$$

A is Singular

A^{-1} does not exist.

$$\begin{vmatrix} 2 & 4 & 2 \\ 1 & -3 & 0 \\ -5 & 5 & -1 \end{vmatrix} = 2\begin{vmatrix} -3 & 0 \\ 5 & -1 \end{vmatrix} - 4\begin{vmatrix} 1 & 0 \\ -5 & -1 \end{vmatrix} + 2\begin{vmatrix} 1 & -3 \\ -5 & 5 \end{vmatrix}$$

$$= 2(3-0) - 4(-1-0) + 2(5-15)$$

$$= 6 + 4 - 20 = \boxed{-10}$$

Since $|A| \neq 0$, then A^{-1} exist.

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|I| = 1 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \cdot 1 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \cdot 1 \cdot (1-0)$$

$$= \boxed{1}$$

$$|I_n| = 1$$

Cramer's Rule

$$\begin{cases} ax + by = c \\ ex + fy = j \end{cases} \qquad D = \begin{vmatrix} a & b \\ e & f \end{vmatrix}$$

$$D_x = \begin{vmatrix} c & b \\ j & f \end{vmatrix} \qquad D_y = \begin{vmatrix} a & c \\ e & j \end{vmatrix}$$

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}$$

$$\begin{cases} 3x - 2y = 5 \\ 4x + y = 3 \end{cases} \qquad D = \begin{vmatrix} 3 & -2 \\ 4 & 1 \end{vmatrix} = 3-(-8) = \boxed{11} \qquad D_x = \begin{vmatrix} 5 & -2 \\ 3 & 1 \end{vmatrix} = 5-(-6) = \boxed{11}$$

$$x = \frac{D_x}{D} = \frac{11}{11} = 1 \qquad (1, -1) \qquad D_y = \begin{vmatrix} 3 & 5 \\ 4 & 3 \end{vmatrix} = 9-20 = \boxed{-11}$$

$$y = \frac{D_y}{D} = \frac{-11}{11} = -1$$

Solve by Cramer's rule

$$\begin{cases} -5x - 8y = 3 \\ 4x + 7y = 13 \end{cases}$$

$$D = \begin{vmatrix} -5 & -8 \\ 4 & 7 \end{vmatrix} = -35 - (-32) = \boxed{-3}$$

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D}$$

$$D_x = \begin{vmatrix} 3 & -8 \\ 13 & 7 \end{vmatrix} = 21 - (-104) = \boxed{125}$$

$$x = \frac{125}{-3} \qquad y = \frac{-77}{-3}$$

$$D_y = \begin{vmatrix} -5 & 3 \\ 4 & 13 \end{vmatrix} = -65 - 12 = \boxed{-77}$$

$$\left\{\left(\frac{-125}{3}, \frac{77}{3}\right)\right\}$$

Whenever $D = 0$,

Do not use Cramer's rule

$$\begin{cases} x + y - z = 0 \\ x - y + z = 4 \\ \quad\; y - z = -3 \end{cases}$$

Solve for z by Cramer's rule.

$$D = \begin{vmatrix} \boxed{1 \quad 1 \quad -1} \\ 1 \quad -1 \quad 1 \\ 0 \quad 1 \quad -1 \end{vmatrix} \qquad D_z = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -1 & 4 \\ 0 & 1 & -3 \end{vmatrix}$$

$$D = 1\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} + (-1)\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix}$$

$$= 1(0) - 1(-1) - 1(1) = 0$$

cannot use Cramer's Rule

Solve for y only by Cramer's rule

$$\begin{cases} x - y = -2 \\ x + z = 4 \\ y - z = 0 \end{cases}$$

$$D = \begin{vmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix} = -2$$

$$y = \frac{D_y}{D} = \frac{-6}{-2} = 3$$

$$D_y = \begin{vmatrix} 1 & -2 & 0 \\ 1 & 4 & 1 \\ 0 & 0 & -1 \end{vmatrix} = -6$$

$$\boxed{y = 3}$$

Evaluate

$$\begin{vmatrix} x & y & 1 \\ 0 & 3 & 1 \\ 2 & 0 & 1 \end{vmatrix} = 0$$

Expand by first row

$$x\begin{vmatrix} 3 & 1 \\ 0 & 1 \end{vmatrix} - y\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} + 1\begin{vmatrix} 0 & 3 \\ 2 & 0 \end{vmatrix} = 0$$

$$\boxed{3x + 2y = 6}$$

this line contains (0,3) & (2,0)

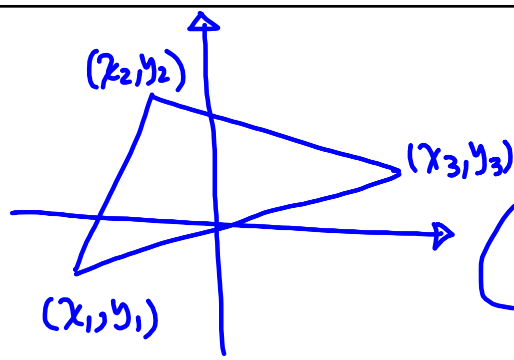
Find eqn of a line that contains $(5,-2)$ & $(2,3)$.

$$\begin{vmatrix} x & y & 1 \\ 5 & -2 & 1 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$$x\begin{vmatrix} -2 & 1 \\ 3 & 1 \end{vmatrix} - y\begin{vmatrix} 5 & 1 \\ 2 & 1 \end{vmatrix} + 1\begin{vmatrix} 5 & -2 \\ 2 & 3 \end{vmatrix} = 0$$

$$-5x - 3y + 19 = 0$$

$$\boxed{5x + 3y = 19}$$

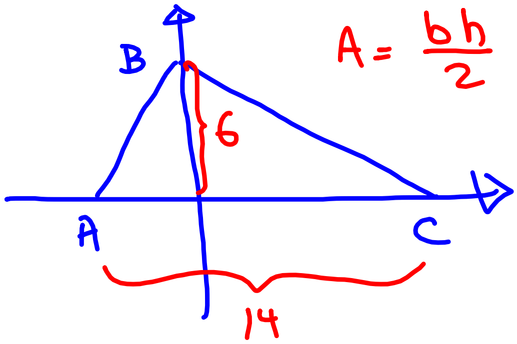


$$A = \pm \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since Area is Positive, use $\pm$ accordingly.

Consider triangle ABC where

A(-4,0), B(0,6), C(10,0) Find its Area.



$A = \frac{bh}{2} = \frac{14 \cdot 6}{2} = \boxed{42}$

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} -4 & 0 & 1 \\ 0 & 6 & 1 \\ 10 & 0 & 1 \end{vmatrix}$$

$$= \pm\frac{1}{2}\left\{-4\begin{vmatrix} 6 & 1 \\ 0 & 1 \end{vmatrix} - 0\begin{vmatrix} \text{who cares!} \end{vmatrix} + 1\begin{vmatrix} 0 & 6 \\ 10 & 0 \end{vmatrix}\right\}$$

$$= \pm\frac{1}{2}\left\{-4(6) - 0 + 1(-60)\right\} = \pm\frac{1}{2}(-84)$$

$$= \boxed{42}$$

Open notes Quiz

① $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -1 \\ 2 & -2 \\ 3 & -3 \end{bmatrix}$ Find AB

② $A = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$ Find A^{-1}